Statistical systems of particles with scalar interaction in cosmology

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Abstract

Cosmological solutions of the equations with scalar interaction are being studied. It is shown, that the scalar field can effectively change the equation of state of statistical system, that leads to series of cosmological consequences.

1 Statistical systems of particles with scalar interaction

Statistical systems of particles with scalar interaction were considered for the first time by one of the Authors. In particular, in Ref. [1], [2] is shown, that the correct insertion of interparticle scalar interaction in the kinetic theory leads to the change of particles effective masses:

$$m_* = |m + q\Phi|, \tag{1}$$

where q - a scalar charge of particles, Φ - potential of scalar field.

1.1 Hamilton description formalism of the particles movement in the scalar field

The canonical equations of the relativistic particles motions concerning to the pair of canonically conjugated dynamic variables x^i (coordinates) and P_i (the generalized momentum) take the form of (see, for example, [1]):

$$\frac{dx^i}{ds} = \frac{\partial H}{\partial P_i}; \qquad \frac{dP_i}{ds} = -\frac{\partial H}{\partial x^i}, \tag{2}$$

where $H(x, P)$ -relativistically invariant Hamilton function. Calculating full derivative of the dynamic variables function $\Psi(x^i, P_k)$, in view of (2) we shall find:

$$\frac{d\Psi}{ds} = [H, \Psi], \tag{3}$$

where invariant Poisson brackets are introduced:

$$[H, \Psi] = \frac{\partial H}{\partial P_i}\frac{\partial \Psi}{\partial x^i} - \frac{\partial H}{\partial x^i}\frac{\partial \Psi}{\partial P_i}. \tag{4}$$

In consequence of (4) Hamilton function is the integral of particle movements, - this integral of movement is called the rest-mass of particles:

$$\frac{dH}{ds} = [H, H] = 0, \Rightarrow H = \frac{1}{2}m^2 = \text{Const.} \tag{5}$$

The relation (6) is called the relation of normalization. Relativistically-invariant Hamilton function particles with scalar charge q, being in a scalar field with potential Φ, is [1]:

$$H(x, P) = \frac{1}{2}m \left[\frac{(P, P)}{m + q\Phi} - q\Phi \right],$$ (6)

where $(a, b) = g_{ik}a^i b^k$-scalar product.

The usual kinematic particle momentum, p^i, is determined through Hamilton function by means of the relation:

$$p^i = \frac{\partial H}{\partial P_i} \quad (= \frac{dx^i}{ds}).$$ (7)

Hence, taking into account (6), we'll receive connection between generalized and kinematic particle momentums:

$$p^i = \frac{P^i}{1 + \dfrac{q\Phi}{m}}.$$ (8)

In consequence of (6) following relations of momentum normalization are carried out:

$$(P, P) = (m + q\Phi)^2; \Rightarrow (p, p) = m^2,$$ (9)

whence it follows that the generalized P_i and kinematic p^i momentums are timelike vectors; in further we'll term the value

$$m_* = m + q\Phi$$ (10)

as an effective mass of a particle. Its meaning is shown in equations of motion (2), which concerning to Hamilton function (6) become:

$$\left(1 + \frac{q\phi}{m}\right)\left[\frac{d^2 x^i}{ds^2} + \Gamma^i_{jk}\frac{dx^j}{ds}\frac{dx^k}{ds}\right] =$$

$$= q\Phi_{,k}\left(g^{jk} - \frac{1}{m^2}\frac{dx^j}{ds}\frac{dx^k}{ds}\right).$$ (11)

The right side (11) is a 4-vector of force F^i, forcing on the particle from the direction of scalar field orthogonally to its velocity vector:

$$(F, p) = 0.$$ (12)

Conversion to a usual normalization of velocity vector

$$u^i = \frac{dx^i}{ds_0},$$

so that

$$(u, u) = 1,$$

is proceeded by renormalization of an interval:

$$s = \frac{s_0}{m}. \tag{13}$$

In terms s_0 equations of motion take more usual form:

$$\left(1 + \frac{q\phi}{m}\right)\left[\frac{d^2 x^i}{ds_0^2} + \Gamma^i_{jk}\frac{dx^j}{ds_0}\frac{dx^k}{ds_0}\right] =$$

$$= q\Phi_{,k}\left(g^{jk} - \frac{dx^j}{ds_0}\frac{dx^k}{ds_0}\right).$$

At availability of space and field symmetry along the vector field $\xi^i(x)$ (Killing field)

$$\underset{\xi}{L}\, g_{ik} = 0; \qquad \underset{\xi}{L}\, \Phi = 0, \tag{14}$$

where $\underset{\xi}{L}\, \Psi$ - Lie derivative of object Ψ along a vector field ξ, the canonical equations of motion (2) or (13) admit linear integrals of motion (see, for example, [5], [6]):

$$\varphi = (\xi, P) = \text{Const}, \tag{15}$$

that make sense of energy and moment integrals.

1.2 Distribution function and macroscopic currents of systems of particles with scalar interaction

Let $F(x, P)$ - invariant distribution function of particles in a 8-dimensional phase space. We shall define the momentums concerning distribution $F(x, P)$ [3]:

$$n^i(x) = \int\limits_{P(x)} p^i dP, \tag{16}$$

number density vector of particles, so that:

$$n^i = nv^i, \tag{17}$$

where v^i-timelike identity vector of kinematic macroscopic velocity of particles:

$$n = \sqrt{(n, n)}; \tag{18}$$

and

$$T_p^{ik}(x) = \left(1 + \frac{q\phi}{m}\right)\int\limits_{P(x)} F(x, P)p^i p^k dP, \tag{19}$$

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- macroscopic momentum-energy tensor (MET) of particles in the universal units system ($G = \hbar = c = 1$). An invariant element of volume of 4-dimensional momentum space in expressions (16), (19) in same units is:

$$dP = \frac{2S+1}{(2\pi)^3\sqrt{-g}}dP_1 \wedge dP_2 \wedge dP_3 \wedge dP_4. \tag{20}$$

The invariant 8-dimensional distribution function
$F(x,P)$ is connected with 7-dimensional distribution function $f(x,P)$ using δ-functions by relationship:

$$F(x,P) = f(x,P)\delta(H - \frac{1}{2}m^2). \tag{21}$$

Solving (20) with the plus value of an energy, we shall obtain an invariant volume unit of 3-dimensional momentum space:

$$dP_+ = \left|1 + \frac{q\Phi}{m}\right| \frac{2S+1}{(2\pi)^3\sqrt{-g}} \frac{dP_1 \wedge dP_2 \wedge dP_3}{|P^4|}, \tag{22}$$

or in terms of kinematic momentum:

$$dP_+ = \left|1 + \frac{q\Phi}{m}\right|^3 \frac{(2S+1)\sqrt{-g}}{(2\pi)^3} \frac{dp^1 \wedge dp^2 \wedge dp^3}{|p^4|}. \tag{23}$$

The sign of absolute value in the above mentioned expressions appears as a result of properties of δ-functions. It is necessary to note, that the form (MET) (19), received for scalar charged particles in Ref.[3], is the unique consequence of the supposition about total momentum conservation during local particles collisions:

$$\sum_I P_i = \sum_F P_k,$$

where summing is carried out by all initial and final states.

Let following responses flow past in system:

$$\sum_{B=1}^{m'} \nu'_B a'_B \rightleftarrows \sum_{A=1}^{m} \nu_A a_A, \tag{24}$$

where a_A - symbols of particles, and ν_A - their numbers. Then

$$P_I = \sum_{B=1}^{m'} \sum_{\alpha'}^{\nu'_B} P_B^{\alpha'},$$

$$P_F = \sum_{A=1}^{m} \sum_{\alpha}^{\nu_A} P_A^{\alpha},$$

and distribution functions of particles are described by invariant kinetic equations [3]:

$$[H_a, f_a] = I_a(x, P_a), \tag{25}$$

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where $J_a(x, P_a)$ - a collision integral:

$$I_a(x, P_a) = -\sum \nu_a \int_a' \delta^4(P_F - P_I) \times$$

$$W_{IF}(Z_{IF} - Z_{FI}) \prod_{I,F}' dP; \tag{26}$$

$$W_{FI} = (2\pi)^4 |M_{IF}|^2 2^{-\sum \nu_A + \sum \nu_b'} -$$

- transition matrix ($|M_{IF}|$ - invariant amplitudes of scattering);

$$Z_{IF} = \prod_I f(P_A^\alpha) \prod_F [1 \pm f(P_B^{\alpha'})];$$

$$Z_{FI} = \prod_I [1 \pm f(P_A^\alpha)] \prod_F f(P_B^{\alpha'}),$$

"+" corresponds to bosons, "- " - to fermions; details see in Ref.[1], [2].

1.3 The self-consistent system of equations for particles with scalar interaction

On the basis of kinetic theory in Ref.[3] there was obtained the self-consistent system of equations describing the statistical self-gravitating system of particles with scalar interaction. We shall define according to Ref.[3] MET of massive scalar field in most general view (with conformally invariant component and cubic nonlinearity):

$$T_s^{ik} = \frac{\epsilon}{8\pi} \left[\frac{4}{3} \Phi^{,i} \Phi^{,k} - \frac{1}{3} g^{ik} \Phi^{,j} \Phi_{,j} + g^{ik} \mu_s^2 \Phi^2 + \right.$$

$$+ \frac{1}{3} \left(Rik - \frac{1}{2} Rgik \right) \Phi^2 - \frac{2}{3} \Phi\Phi, ik+ \tag{27}$$

$$\left. + \frac{2}{3} gik\Phi\Box\Phi + \frac{\lambda}{6} gik\Phi^4 \right],$$

where $\epsilon = +1$ in case of scalar interaction with an attraction of likely scalar charged particles, $\epsilon = -1$ - for repulsion - of likely charged particles. Also by means of distribution function we'll define a scalar $\sigma(x)$ [3]:

$$\sigma(x) = \sum m_A q_A \int_{P(x)} dP_A F_A(x, P_A). \tag{28}$$

The introduced scalar can be expressed through the spur of MET particles:

$$\sigma(x) = \sum_A \frac{q_A T_p^A}{m_A + q_A \Phi}, \tag{29}$$

where T_p - spur of MET particles. Further we shall term $\sigma(x)$ as a *scalar density of charges*. It is necessary to note, that the scalar density of charges is unambiguously determined by conservation laws. Thus, Einstein's equations for statistical systems of scalar charged particles take form:

$$R^{ik} - \frac{1}{2}Rg^{ik} = 8\pi(T_p^{ik} + T_s^{ik}),\tag{30}$$

and the equation of a scalar field with a source (29) is:

$$\Box\Phi + \mu_s^2\Phi - \frac{1}{6}R\Phi + \frac{\lambda}{3}\Phi^3 = -4\pi\epsilon\sigma.\tag{31}$$

The system of equations (25), (30), (31) together with definitions (19), (27) and (29) represent the required closed system of the self-consistent equations, describing the statistical system of particles with scalar interaction.

1.4   The local thermodynamic equilibrium

The local thermodynamic equilibrium (LTE) is reached in statistical system, when a medial run length (time between collisions) is much less than the typical scale of inhomogeneity of system (the typical time of evolution). In that cases the integral of collisions is a main term in the kinetic equations that leads to so-called functional equations of Boltzmann, having as the solutions a locally-equilibrium distribution functions:

$$f^0(x,P) = \left\{\exp\left[\frac{-\mu_a + (v,P_a)}{\theta}\right] \mp 1\right\}^{-1},\tag{32}$$

where the upper sign corresponds to bosons, lower sign - to fermions, $\theta(x)$ - local temperature, identical to all sorts of particles, v^i - identity timelike macroscopic velocity vector of statistical system, $\mu_a(x)$ - chemical potentials, satisfying the system of linear algebraic equations of a chemical equilibrium (according to responses (24):

$$\sum_{B=1}^{m'} \nu'_B\mu_B = \sum_{A=1}^{m} \nu_A\mu_A.\tag{33}$$

Macroscopic characteristics $\theta(x)$, $v^i(x)$, $\mu_a(x)$ are defined from a self-consistent system of equations (30), (31), (33) and definitions (19), (27) and (29). Thus we shall obtain [6]:

$$n^i(x) = v^i\frac{2S+1}{2\pi^2}\int\limits_0^\infty p^2 dp\times$$

$$\times\left\{\exp\left[\frac{-\mu_a + \sqrt{m_*^2 + p^2}}{\theta}\right] \mp 1\right\}^{-1}\tag{34}$$

$$T_a^{ik}(x) = [\mathcal{E}_a(x) + P_a(x)]v^i v^k - P_a(x)g^{ik} \tag{35}$$

where introduced: scalar density of energy

$$\mathcal{E} = \sum_a \mathcal{E}_a$$

and pressure of system of particles:

$$P = \sum_a P_a :$$

$$\mathcal{E}_a(x) = \frac{2S+1}{2\pi^2} \int\limits_0^\infty dp \cdot p^2 \sqrt{m_*^2 + p^2} \times$$

$$\times \left\{ \exp\left[\frac{-\mu_a + \sqrt{m_*^2 + p^2}}{\theta}\right] \mp 1 \right\}^{-1}, \tag{36}$$

$$P_a(x) = \frac{2S+1}{6\pi^2} \int\limits_0^\infty \frac{dp \cdot p^4}{\sqrt{m_*^2 + p^2}} \times$$

$$\times \left\{ \exp\left[\frac{-\mu_a + \sqrt{m_*^2 + p^2}}{\theta}\right] \mp 1 \right\}^{-1}. \tag{37}$$

Let's also define an equilibrium density of a scalar charge:

$$\sigma(x) = \sum \frac{q(2S+1)m_*}{2\pi^2} \int\limits_0^\infty \frac{dp \cdot p^2}{\sqrt{m_*^2 + p^2}} \times$$

$$\times \left\{ \exp\left[\frac{-\mu_a + \sqrt{m_*^2 + p^2}}{\theta}\right] \mp 1 \right\}^{-1}. \tag{38}$$

The differential consequence of the Einstein equations in the case of LTE are equations of ideal hydrodynamics, which in the case of scalar interaction can be lead to the form [6]:

$$\nabla_i[\mathcal{E} + P)v^i] - (P_{,i} + \sigma\Phi_{,i})v^i = 0, \tag{39}$$

- the continuity equation for energy, (∇_i - symbol covariant derivative) and

$$\nabla_i[\mathcal{E} + P)v^i] - (P_{,i} + \sigma\Phi_{,i})v^i = 0, \tag{40}$$

- equations of motion.

Besides, if any vectorial charges conserve (for example, electrical), we have the additional equations of continuities for corresponding currents:

$$\nabla_i(v^i \sum_a e_a n_a) = 0, \tag{41}$$

where e_a - corresponding vector charges.

2 Completely degenerate Fermi-gas with scalar interaction

In this article we shall consider completely degenerate one-sortable Fermi-gas consisting from massive particles with a spin $1/2$ as a concrete statistical system. The full degeneration condition supposes

$$\frac{\mu}{\theta} \to \infty. \tag{42}$$

In this case the locally-equilibrium distribution function has a form:

$$f^0(x, P) = \begin{cases} 0, & \mu \leqslant \sqrt{m_*^2 + p^2}; \\ \\ 1, & \mu > \sqrt{m_*^2 + p^2} \end{cases}. \tag{43}$$

Therefore the integration of macroscopic densities is representable in elementary functions:

$$\mathcal{E} = \frac{m_*^4}{8\pi^2} \left[\psi\sqrt{1 + \psi^2}(1 + 2\psi^2) - \ln(\psi + \sqrt{1 + \psi^2}) \right];$$

$$P = \frac{m_*^4}{24\pi^2} \left[\psi\sqrt{1 + \psi^2}(2\psi^2 - 3) + 3\ln(\psi + \sqrt{1 + \psi^2}) \right];$$

$$T = \mathcal{E} - 3P = \frac{m_*^4}{2\pi^2} \left[\psi\sqrt{1 + \psi^2} - \ln(\psi + \sqrt{1 + \psi^2}) \right],$$

$$\mathcal{E} + P = \frac{m_*^4}{3\pi^2} \psi^3 \sqrt{1 + \psi^2}, \tag{44}$$

$$\sigma = \frac{q \cdot m_*^3}{2\pi^2} \left[\psi\sqrt{1 + \psi^2} - \ln(\psi + \sqrt{1 + \psi^2}) \right], \tag{45}$$

where $\psi = p_F/|m_*|$ - the ratio of Fermi momentum to effective mass. In that case the self-consistent equation of massive scalar field becomes [1]:

$$\Box\Phi + \mu^2\Phi = -\frac{4\pi}{(m + q\Phi)^2} qT,$$

and a density of number of fermions with Fermi momentum is connected by relationship Ref.[7]:

$$n(x) = \frac{p_F^3}{3\pi^2} \Rightarrow p_F = (3\pi^2 n(x))^{\frac{1}{3}}. \tag{46}$$

Thus, the variable ξ can be expressed through two scalars - density of particles number in the natural frame of reference and scalar potential:

$$\psi = \frac{(3\pi^2 n(x))^{\frac{1}{3}}}{|m + q\Phi|}. \tag{47}$$

3 Static degenerated Fermi-system

Let's consider at first a global thermodynamic equilibrium of rest degenerate gas, which is possible at (Ref.[3], [5]):

$$\underset{\xi}{L}\, g_{ik} = 0; \qquad \underset{\xi}{\Phi} = 0, \tag{48}$$

and ξ^i is - timelike vector. In that case equilibrium distribution functions are exactly solutions of kinetic equations, where it is necessary to specialize a coordinate association of field magnitudes and thermodynamic parameters (Ref.[3]):

$$v^i = \frac{\xi^i}{\sqrt{\xi,\xi}}; \quad \theta = \theta_0 \sqrt{\xi,\xi}, \quad \mu = \mu_0 \sqrt{\xi,\xi}. \tag{49}$$

Supposing

$$\xi^i = \delta_4^i,$$

we shall obtain:

$$\partial_t g_{ik} = 0; \quad \partial_t \Phi = 0; \quad \mu = \mu_0 \sqrt{g_{44}}, \tag{50}$$

Let further

$$g_{44}(\infty) = 1, \qquad \Phi(\infty) = \Phi_0, \quad p_F(\infty) = p_F^0.$$

That way we will find:

$$\psi = \frac{\sqrt{\rho^2 - (1 + q\Phi/m)^2}}{(1 + q\Phi/m)}, \tag{51}$$

where the dimensionless function is introduced

$$\rho = \frac{\sqrt{g_{44}(m_0^2 + (p_F^0)^2)}}{m_0} \geqslant 1$$

and

$$m_0 = m + q\Phi_0.$$

Introducing further a new dimensionless field variable ξ, so that:

$$1 + \frac{q\Phi}{m} = \rho\xi \Rightarrow q\Phi = m(\rho\xi - 1) \Rightarrow \psi = \frac{\sqrt{1 - \xi^2}}{|\xi|}, \tag{52}$$

we will obtain:

$$\mathcal{E}_f =$$

$$\frac{m^4\rho^4}{8\pi^2}\left[(2 - \xi^2)\sqrt{1 - \xi^2} - \xi^4 \ln \frac{\sqrt{1 - \xi^2} + 1}{|\xi|}\right]; \tag{53}$$

$$P_f =$$

$$\frac{m^4\rho^4}{24\pi^2}\left[(2 - 5\xi^2)\sqrt{1 - \xi^2} - \xi^4 \ln \frac{\sqrt{1 - \xi^2} + 1}{|\xi|}\right]; \tag{54}$$

$$\sigma = q\frac{m^3\rho^3}{2\pi^2}\left[\sqrt{1-\xi^2} - \xi^2\ln\frac{\sqrt{1-\xi^2}+1}{|\xi|}\right].\tag{55}$$

It is easy to obtain the limiting relations for these magnitudes in case of

$$1 + \frac{q\Phi}{m} = 0 \Rightarrow \xi = 0:\tag{56}$$

$$\lim_{\xi\to 0}\mathcal{E}_f = \frac{m^4\rho^4}{4\pi^2};\tag{57}$$

$$\lim_{\xi\to 0}P_f = \frac{m^4\rho^4}{12\pi^2} = \frac{1}{3}\mathcal{E};\tag{58}$$

$$\lim_{\xi\to 0}\sigma = q\frac{m^3\rho^3}{2\pi^2},\tag{59}$$

Thus, in a limit (56) system of degenerate fermions becomes ultrarelativistic.

At the given gravitational field macroscopic characteristics of plasma appear explicitly depending only from the potential scalar field, and the self-consistent equations of globally equilibrium completely degenerate fermion system with scalar interaction are described only by one self-consistent equation of type of Klein-Gordon:

$$\Box\Phi + \mu_s^2\Phi - \frac{1}{6}R\Phi + \frac{\lambda}{3}\Phi^3 + \frac{R}{6}\Phi =$$

$$-4\pi\epsilon q\frac{m^3\rho^3}{2\pi^2}\left[\sqrt{1-\xi^2} - \xi^2\ln\frac{\sqrt{1-\xi^2}+1}{|\xi|}\right]\tag{60}$$

In this case the density of potential energy of scalar fields at the lack of substance is equal:

$$\mathcal{E}_s = \mu^2\Phi^2 + \frac{\lambda}{3}\Phi^4.\tag{61}$$

The interaction energy of scalar field with a substance is obtained from density of the energy of Fermi-system minus the same magnitude at the lack of a scalar field:

$$\mathcal{E}_{in} = \mathcal{E}(\Phi) - \mathcal{E}(0) =$$

$$= \frac{m^4\rho^4}{8\pi^2}\left[(2-\xi^2)\sqrt{1-\xi^2} - \xi^4\ln\frac{\sqrt{1-\xi^2}+1}{|\xi|} - \right.$$

$$\left. - \left(\frac{(2\rho^2-1)\sqrt{\rho^2-1}}{\rho^4} - \frac{1}{\rho^4}\ln\sqrt{\rho^2-1} + \rho\right)\right]\tag{62}$$

Let's discard for simplicity conformally invariant term in the momentum energy tensor of scalar field and suppose $\lambda = 0$; then the summarized potential energy of scalar field with its interaction's energy with Fermi-system is equal:

$$\mathcal{E}_\Sigma = \mathcal{E}_S + \mathcal{E}_{in} = q^2 m^4 \rho^2\left\{\alpha^2(\rho\xi - 1)^2 + \right.$$

$$+\frac{1}{8\pi^2}\left[(2-\xi^2)\sqrt{1-\xi^2}-\xi^4\ln\frac{\sqrt{1-\xi^2}+1}{|\xi|}-\right.$$

$$\left.\left.-\left(\frac{(2\rho^2-1)\sqrt{\rho^2-1}}{\rho^4}-\frac{1}{\rho^4}\ln\sqrt{\rho^2-1}+\rho\right)\right]\right\},\tag{63}$$

where

$$\alpha^2=\frac{\mu^2}{q^2m^4\rho^2}.$$

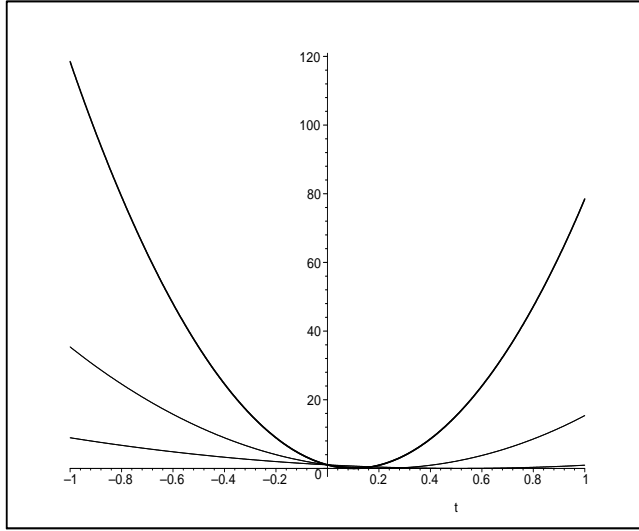In Fig. 1 graphs of potential energy $\mathcal{E}_\Sigma/q^2m^4\rho^2$ are shown.

Figure 1: Dependence graph of energy $\mathcal{E}_\Sigma/q^2m^4\rho^2$. from parameter ρ: (a thin line - $\rho=2$, a medial line - $\rho=5$, bold line - $\rho=10$); everywhere $\alpha=1$.

In Fig. 2 the same graphs in a fine scale are shown - here it is visible that all graphs intersect a line of zero-point energy, and their minimums lay in the subzero region.
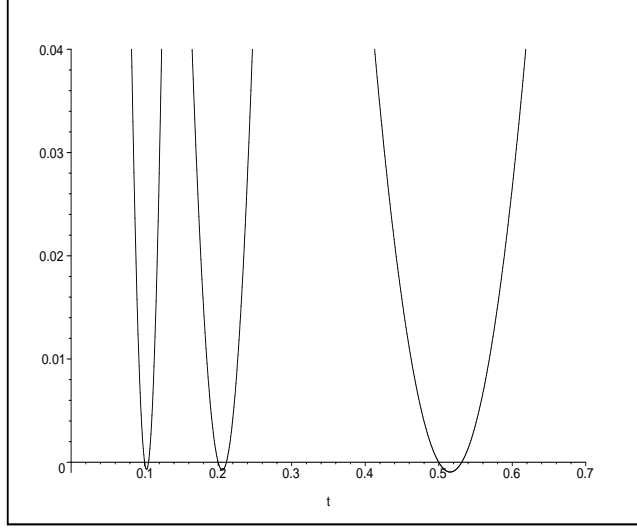
Figure 2: Dependence graph of energy $\mathcal{E}_\Sigma/q^2m^4\rho^2$. from parameter ρ=2,5,10(from left to right); everywhere $\alpha = 1$.

Hence, the account of a scalar field's influence on the statistical system leads to the appearance of non-linear effective potential of interaction which can have minimums at nonzero values Φ, that can perform excessive an operation of artificial introduction of cubic nonlinearity in dynamic equations of scalar field. Besides, the introduction of scalar interactions leads to the effective mechanism of state equations regulation, which can appear important in cosmological situation.

4 Homogeneous degenerate Fermi-system in cosmology

Let's consider a cosmological situation when the substance is presented only as degenerate Fermi-system with scalar interaction of particles. In this case the self-consistent system of Einstein and Klein-Gordon equations with a scalar source in the metric

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2) \tag{64}$$

takes form:

$$3\frac{\dot{a}^2}{a^2} = 8\pi \tag{65}$$

In this metric

$$v^i = \delta_4^i$$

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and from the relation (46) and conservation law of particles:

$$\partial_i \sqrt{-g}nv^i = 0$$

we will obtain the momentum integral instead of the energy integral (50):

$$ap_F = \text{Const.} \tag{66}$$

Supposing that

$$\Phi = \Phi(t); \Rightarrow \mathcal{E} = \mathcal{E}(t); \; P = P(t),$$

we shall obtain the structure of summary momentum-energy tensor of scalar field in the form of momentum-energy tensor of ideal fluid with macroscopic velocity v^i and a density of energy $\mathcal{E}_S$ and pressure P_S:

$$\mathcal{E}_s = \frac{1}{8\pi}(\dot{\Phi}^2 + \mu_s^2\Phi^2); \quad P_s = \frac{1}{8\pi}(\frac{1}{3}\dot{\Phi}^2 - \mu_s^2\Phi^2). \tag{67}$$

As it's known, (see, for example, [8]), in the metric (64) Einstein independent equations have a form:

$$3\frac{\dot{a}^2}{a^2} = 8\pi\varepsilon; \tag{68}$$

$$3\frac{\dot{a}}{a} = -\frac{\dot{\varepsilon}}{\varepsilon + P}. \tag{69}$$

According to the momentum integral (66) let's introduce new dimensionless variables and parameters:

$$\varphi = \frac{1}{\psi}; \quad \beta = \frac{p_F^0}{m}. \tag{70}$$

Then:

$$\Phi = \frac{m}{q}\left(\frac{\beta\varphi}{a} - 1\right), \quad m_* = m\beta\frac{\varphi}{a}, \tag{71}$$

where we have supposed:

$$p_F^0 = p_F(t_0); a(t_0) = 1.$$

Thus, it remains two unknown functions, $a(t)$ and $\psi(t)$.

As the equation of a scalar field is a consequence of Einstein equations it can be missed. Considering an explicit structure of expressions for a density of energy and pressure,let's introduce for convenience conformal densities according to the rule:

$$X^* = a^4X. \tag{72}$$

Einstein equations for conformal densities have a form:

$$\mathcal{E}^{*\,\prime} - \frac{a'}{a}(\mathcal{E}^* - 3P^*) = 0; \tag{73}$$

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$$3a'^2 = 8\pi \mathcal{E}^*, \tag{74}$$

where we have changed over to the temporal variable derivation η by rule:

$$ad\eta = dt \Rightarrow \dot{X} = \frac{X'}{a};$$

- $X' = dX/d\eta$. In new variables densities of energy and pressure of Fermi-system and scalar field can be written in the form of:

$$\mathcal{E}_f^* = \frac{m^4\beta^4}{8\pi^2} \times$$

$$\times \left[\sqrt{1+\varphi^2}(2+\varphi^2) - \varphi^4 \ln \frac{1+\sqrt{1+\varphi^2}}{\varphi} \right]; \tag{75}$$

$$\mathcal{E}_f^* - 3P_f^* =$$

$$\frac{m^4\beta^4}{3\pi^2}\varphi^2\left(\sqrt{1+\varphi^2} - \varphi^2 \ln \frac{1+\sqrt{1+\varphi^2}}{\varphi}\right); \tag{76}$$

$$\mathcal{E}_s^* = \frac{\beta^2 m^2}{8\pi q^2} \left[\varphi^2 - 2\varphi\varphi' + \varphi'a + \mu^2 a^2 \left(\varphi - \frac{a}{\beta}\right) \right]; \tag{77}$$

$$\mathcal{E}_s^* - 3P_s^* = \frac{\mu_s^2 \Phi^2}{2\pi} = \frac{\beta^2 \mu_s^2 m^2}{2\pi q^2} a^2 \left(\varphi - \frac{a}{\beta}\right)^2. \tag{78}$$

Calculating, we'll find:

$$\varphi \frac{d\mathcal{E}_f^*}{d\varphi} = \frac{m^4\beta^4}{2\pi^2}\varphi^2 \times$$

$$\left[\sqrt{1+\varphi^2} - \varphi^2 \ln\left(\frac{1+\sqrt{1+\varphi^2}}{\varphi}\right) \right].$$

$$\varphi \frac{d\mathcal{E}_f^*}{d\varphi} = \frac{3}{2}(\mathcal{E}_f^* - 3P_f^*) \tag{79}$$

Substituting the obtained expressions in the equations (77), (78), we will obtain the system of two ordinary differential equations of the first and second order concerning variables $a(\eta)$ and $\varphi(\eta)$. Below some results of numerical integration of this system are represented.
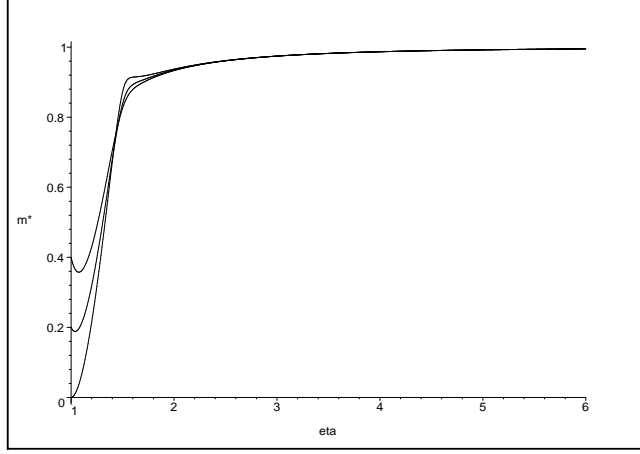
Figure 3:   Evolution of the effective mass m_* of fermions depending on initial mass.
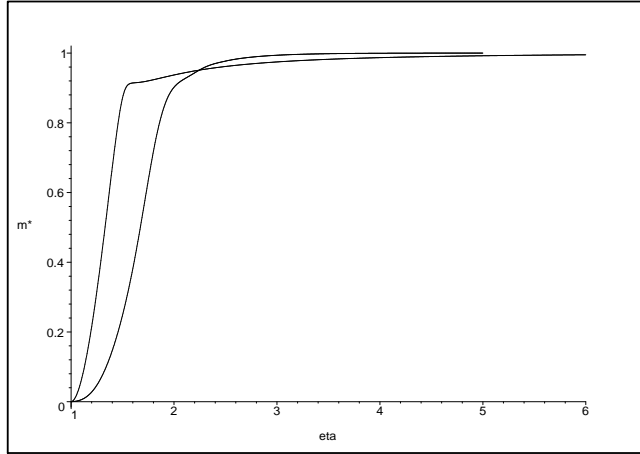

Figure 4:   Evolution of the effective mass m_* fermions depending on parameter $\rho = 2$-thin line and $\beta = 10$ - bold line.
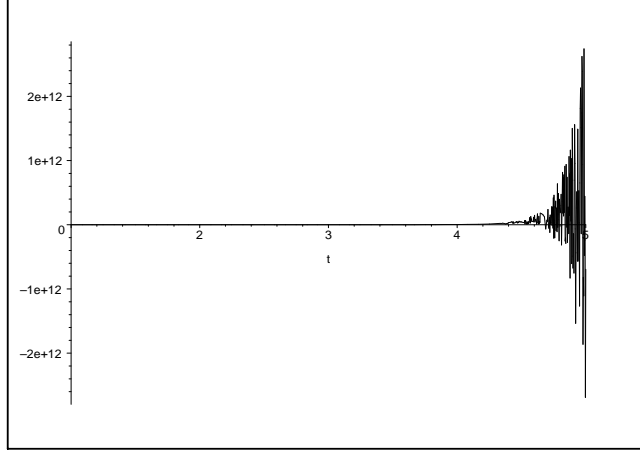
15

Figure 5: *Evolution of a track of conformal MET $\mathcal{E}^* - 3P^*$. The singularity is visible near $\eta \sim 5$.*

Conclusion

Mentioned above results of numerical integration of Einstein - Klein - Gordon equations for degenerate Fermi-gas of scalar charged particles exhibit an essential influence of Fermi-system interaction with scalar field on the summary equation system state. Detection of this influence by means of numerical modelling of system will be considered in our following article.

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